

Quintessence Model and Observational Constraints

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Abstract

The recent observations of type Ia supernovae strongly support that the universe is accelerating now and decelerated in the recent past. By assuming a general relation between the quintessence potential and the quintessence kinetic energy, a general relation is found between the quintessence energy density and the scale factor. The potential includes both the hyperbolic and the double exponential potentials. A detailed analysis of the transition from the deceleration phase to the acceleration phase is then performed. We show that the current constraints on the transition time, the equation of state and the energy density of the quintessence field are satisfied in the model.

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The recent observations of Type Ia supernovae strongly support that the expansion of the Universe is accelerating [1][2]. The observation of type Ia supernova SN 1997ff at $z \sim 1.7$ also provides the evidence of a decelerated universe in the recent past [3]. Turner and Riess showed that the supernova data favored recent acceleration ($z < 0.5$) and past deceleration ($z > 0.5$) [4]. On the other hand, the measurement of the acoustic peaks in the angular power spectrum of the cosmic microwave background supports a flat universe as predicted by the inflationary models [5][6]. One immediate conclusion is that the universe is dominated by a form of matter with negative pressure now, this form of matter is widely referred as dark energy. One simple candidate of the dark energy is the cosmological constant. However, there are some problems although cold dark matter cosmological constant models are consistent with the current observations. Why does the vacuum energy begin to dominate presently? This problem is referred as the coincident problem. Why is the cosmological constant so small and not zero? The quintessence models avoid these problems [7]-[24]. The basic idea of the quintessence model bases upon a scalar field Q that slowly evolves down its potential $V(Q)$. The slowly evolved scalar field takes the role of a dynamical cosmological constant. The energy density of the scalar field must remain very small compared with radiation and matter at early epoches and evolves in a way that it started to dominate the universe at very recent past. This requires some constraints on the initial conditions and fine tuning of the potential. A tracker field was used to solve the initial condition and fine tuning problems. The missing energy was comparable to the radiation energy at the very early time, tracked the background energy density for most of the history of the universe, and then grew to dominate the energy density recently so that the Universe is accelerating now. Although these models successfully explain the dark energy of the universe, they fail to discuss matching the quintessence field dominating the accelerating universe with the matter dominating the decelerating universe. Sen and Sethi addressed the smoothly matching issue by assuming a particular form of the scale factor [25]. In this paper, we assumed a type of potential that has a general relation with the kinetic term of the quintessence field and then obtained an exact relation between the energy density of the quintessence field and the scale factor. This type of solution includes the constant pressure solution corresponding to the double exponential potential given in [25] and the constant equation of state solution corresponding to hyperbolic potential discussed in [26][27]. The constraints on the current values of ω_Q and Ω_Q given in [28] were then used to show that the model passed those

constraints. Those constraints disfavored some quintessence models, like the inverse power law potential [20][22][23].

For a spatially flat, isotropic and homogeneous universe with both an ordinary pressureless dust matter and a minimally coupled scalar field Q sources, the Friedmann equations are

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_Q), \quad (1)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_m + \rho_Q + 3p_Q), \quad (2)$$

$$\ddot{Q} + 3H\dot{Q} + V'(Q) = 0, \quad (3)$$

where dot means derivative with respect to time, $\rho_m = \rho_{m0}(a_0/a)^3$ is the matter energy density, a subscript 0 means the value of the variable at present time, $\rho_Q = \dot{Q}^2/2 + V(Q)$, $p_Q = \dot{Q}^2/2 - V(Q)$, $V'(Q) = dV(Q)/dQ$ and $V(Q)$ is the potential of the quintessence field. To proceed, we assume the following general relationship

$$V(Q) = \beta\dot{Q}^2 + C, \quad (4)$$

instead of assuming a particular potential for the quintessence field or a particular form of the scale factor, where β and C are constants. The above general potential includes the hyperbolic potential and the double exponential potential. From Eq. (4), we get

$$V'(Q) = 2\beta\ddot{Q}. \quad (5)$$

Substitute Eq. (5) to Eq. (3), we get

$$\dot{Q}a^{3/(2\beta+1)} = C_2, \quad (6)$$

where C_2 is an integration constant. Therefore, the energy density and pressure of the quintessence field become

$$\rho_Q = \frac{(1/2 + \beta)C_2^2}{a^{6/(2\beta+1)}} + C, \quad (7)$$

$$p_Q = \frac{(1/2 - \beta)C_2^2}{a^{6/(2\beta+1)}} - C. \quad (8)$$

The equation of state of the quintessence field is

$$\omega_Q = \frac{(1/2 - \beta)C_2^2 - Ca^{6/(2\beta+1)}}{(1/2 + \beta)C_2^2 + Ca^{6/(2\beta+1)}}.$$

In terms of ρ_{Q0} and ω_{Q0} , we have

$$C_2^2 = (1 + \omega_{Q0})\rho_{Q0}a^{6/(2\beta+1)}, \quad (9)$$

$$C = \left[\frac{1}{2} - \beta - \left(\frac{1}{2} + \beta \right) \omega_{Q0} \right] \rho_{Q0}, \quad (10)$$

$$\rho_Q = (1/2 + \beta)(1 + \omega_{Q0})\rho_{Q0} \left(\frac{a_0}{a} \right)^{6/(2\beta+1)} + \left[\frac{1}{2} - \beta - \left(\frac{1}{2} + \beta \right) \omega_{Q0} \right] \rho_{Q0}, \quad (11)$$

$$p_Q = (1/2 - \beta)(1 + \omega_{Q0})\rho_{Q0} \left(\frac{a_0}{a} \right)^{6/(2\beta+1)} - \left[\frac{1}{2} - \beta - \left(\frac{1}{2} + \beta \right) \omega_{Q0} \right] \rho_{Q0}, \quad (12)$$

$$H^2 = \frac{8\pi G}{3} \left\{ \rho_{m0} \left(\frac{a_0}{a} \right)^3 + (1/2 + \beta)(1 + \omega_{Q0})\rho_{Q0} \left(\frac{a_0}{a} \right)^{6/(2\beta+1)} + C \right\}. \quad (13)$$

The transition from deceleration to acceleration happens when the deceleration parameter $q = -\ddot{a}H^2/a = 0$. From Eqs. (2), (11) and (12), in terms of the redshift parameter $1 + z = a_0/a$, we have

$$(1 + z_{q=0})^3 + 2(1 - \beta)(1 + \omega_{Q0})\frac{\rho_{Q0}}{\rho_{m0}}(1 + z_{q=0})^{6/(2\beta+1)} - [1 - 2\beta - (1 + 2\beta)\omega_{Q0}]\frac{\rho_{Q0}}{\rho_{m0}} = 0 \quad (14)$$

This equation gives a relationship between ω_{Q0} and Ω_{Q0} . Now let us look at some special cases.

Case I: $C = 0$. This is the case that the equation of state of the scalar field is a constant, $\omega_Q = (1/2 - \beta)/(1/2 + \beta)$. The potential is [26]

$$V(Q) = A[\sinh k(Q/\alpha + B)]^{-\alpha},$$

where $\alpha = 2/(\beta - 1/2)$, $k^2 = 48\pi G/(2\beta + 1)$, $A^{\beta-1/2} = (1/2 + \beta)C_2^{2\beta+1}\beta^{\beta-1/2}/(\rho_{m0}a_0^3)$ and B is an arbitrary integration constant. The energy density of the quintessence field evolves as

$$\rho_Q = \rho_{Q0} \left(\frac{a_0}{a} \right)^{3(1+\omega_{Q0})}.$$

Eq. (14) becomes

$$(1 + z_{q=0})^3 + \frac{\rho_{Q0}}{\rho_{m0}}(1 + 3\omega_{Q0})(1 + z_{q=0})^{3(1+\omega_{Q0})} = 0.$$

Use the observation results $\rho_{Q0}/\rho_{m0} = 7/3$ and $z_{q=0} = 0.5$, we get $\omega_{Q0} = -0.65$. However, if we take $z_{q=0} = 0.666$, then $\omega_{Q0} = -0.9$. If we take $z_{q=0} = 0.5$ and $\rho_{Q0}/\rho_{m0} = 0.64/0.36 = 1.8$, then $\omega_{Q0} = -0.88$. This shows that the transition time $z_{q=0}$ from deceleration to

acceleration, ω_{Q0} and Ω_{Q0} are very sensitive to each other. The recent constraint is $\omega_{Q0} < -0.85$.

Case II: $\beta = 1/2$. This is the case discussed in [25]. This case gives a constant pressure $p_Q = \omega_{Q0}\rho_{Q0}$ for the scalar field and the potential is the double exponential potential. The solutions to Eqs. (11) and (13) are

$$\rho_Q = (1 + \omega_{Q0})\rho_{Q0} \left(\frac{a_0}{a}\right)^3 - \omega_{Q0}\rho_{Q0} = \begin{cases} (1 + \omega_{Q0})\rho_{Q0} \left(\frac{a_0}{a}\right)^3 & a_0/a \gg 1, \\ -\omega_{Q0}\rho_{Q0} & a_0/a \ll 1. \end{cases}, \quad (15)$$

$$\rho_{Q0} + \rho_{m0} = \frac{1}{6\pi G t_0^2} [\coth(1)]^2, \quad (16)$$

$$a(t) = \frac{a_0}{[\sinh(1)]^{2/3}} [\sinh(t/t_0)]^{2/3}. \quad (17)$$

During matter dominated epoch, the quintessence field should be sub-dominated and this gives the constraint $(1 + \omega_{Q0})\rho_{Q0} < \rho_{m0}$. Now Eq. (14) becomes

$$(1 + z_{q=0})^3 + (1 + \omega_{Q0}) \frac{\rho_{Q0}}{\rho_{m0}} (1 + z_{q=0})^3 + 2\omega_{Q0} \frac{\rho_{Q0}}{\rho_{m0}} = 0.$$

Use $z_{q=0} = 0.5$ and $\rho_{Q0}/\rho_{m0} = 7/3$, we get $\omega_{Q0} = -0.897$ and $(1 + \omega_{Q0})\rho_{Q0}/\rho_{m0} = 0.24$. If we use $z_{q=0} \sim 0.67$ or $\rho_{Q0}/\rho_{m0} \sim 0.63/0.37 = 1.7$, then $\omega_{Q0} \sim -1$. Again the transition time $z_{q=0}$ from deceleration to acceleration, ω_{Q0} and Ω_{Q0} are very sensitive to each other.

Case III: $\beta = 1$. The energy density of the scalar field evolves as

$$\rho_Q = \frac{3}{2}(1 + \omega_{Q0})\rho_{Q0} \left(\frac{a_0}{a}\right)^2 - \frac{1}{2}(1 + 3\omega_{Q0})\rho_{Q0}.$$

The evolution of the Universe is the same as that of $k = -1$ with a cosmological constant. The constraint equation (14) becomes

$$(1 + z_{q=0})^3 + (1 + 3\omega_{Q0}) \frac{\rho_{Q0}}{\rho_{m0}} = 0.$$

Take $z_{q=0} = 0.5$ and $\rho_{Q0}/\rho_{m0} = 7/3$, we get $\omega_{Q0} = -0.82$.

It is also interesting to note that for a pure cosmological constant ($C_2 = 0$ in our model), we have the relation

$$(1 + z_{q=0})^3 = \frac{2\rho_{Q0}}{\rho_{m0}}.$$

With the assumption of Eq. (4), we are able to get the energy density Eq. (11) of the quintessence field. The energy density of the quintessence field has the property that it is sub-dominated at higher redshift and becomes dominated at present time. This solution makes

it possible to study the detailed transition from deceleration phase to acceleration phase. The model also gives the relation among $z_{q=0}$, ω_{Q0} and Ω_{Q0} . The constraints $\omega_{Q0} < -0.85$, $\Omega_{Q0} = 0.65 \pm 0.15$ and $z_{q=0} \sim 0.5$ are satisfied in the model. If $\omega_{Q0} \rightarrow -1$, then Ω_{Q0} tends to take lower value or $z_{q=0}$ tends to take higher value. The model can also easily pass the constraint $\Omega_Q(\text{Mev}) < 0.045$.

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